EECS201000
Introduction to Programming Laboratory

Homework 4: Blocked All-Pairs Shortest Path

Due: Aug 3, 2017, 8AM

1 GOAL

This assignment helps you get familiar with CUDA on multi-GPU environment by implementing a blocked all-pairs shortest path algorithm. We encourage you to optimize your program by exploring different optimizing strategies for optimization points.

2 PROBLEM DESCRIPTION

In this assignment, you are asked to modify sequential Floyd-Warshall algorithm to a parallelized CUDA version which take advantages of multiple GPUs.

Given an $N \times N$ matrix $W = [w(i,j)]$ where $w(i,j) \geq 0$ represents the distance (weight of the edge) from a vertex i to a vertex j in a ***simple directed graph*** with N vertices. We define an $N \times N$ matrix $D = [d(i,j)]$ where $d(i,j)$ denotes the shortest-path distance from a vertex i to a vertex j. Let $D^{(k)} = \left[d^{(k)}(i,j)\right]$ be the result which all the intermediate vertices are in the set $\{1,2,\cdots,k\}$.

We define $d^{(k)}(i,j)$ as follows:

$$d^{(k)}(i,j) = \begin{cases} w(i,j) & if\ k = 0; \\ \min\left(d^{(k-1)}(i,j), d^{(k-1)}(i,k) + d^{(k-1)}(k,j)\right) & if\ k \geq 1. \end{cases}$$

The matrix $D^{(N)} = d^{(N)}(i,j)$ gives the answer to the APSP problem.

In the blocked APSP algorithm, we partition D into $\lceil N/B \rceil \times \lceil N/B \rceil$ blocks of $B \times B$ submatrices. The number B is called ***blocking factor***. For instance, we divide a 6×6 matrix into 3×3 submatrices (or blocks) by $B = 2$.

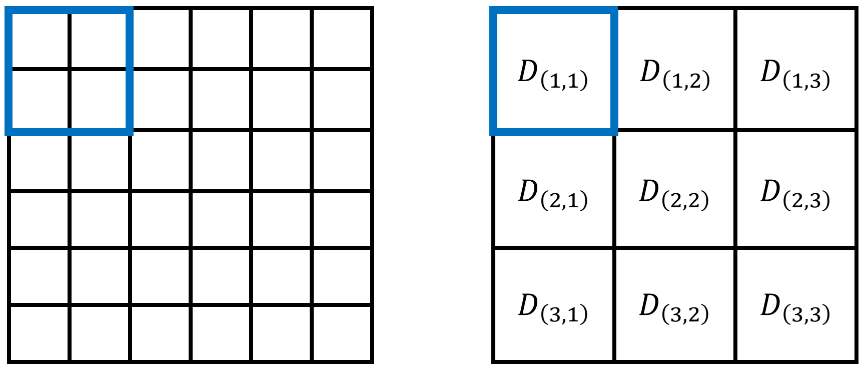


Figure 1: Divide a matrix by $B = 2$

Blocked version of Floyd-Warshall algorithm will perform $\lceil N/B \rceil$ rounds, and each round is divided into 3 phases. It performs B iterations in each phase.

Assumes a block is identified by its index (I, J), where $1 \le I, J \le \lceil N/B \rceil$. The block with index (I, J) is denoted by $D_{(I,J)}^{(k)}$.

In the following explanation, we assumes $N = 6$ and $B = 2$. The execution flow is described step by step as follows:

- **Phase 1:** Self-dependent blocks

  In the K-th iteration, the 1st phase is to compute $B \times B$ pivot block $D_{(K,K)}^{(K \times B)}$.

  For instance, in the 1st iteration, $D_{1,1}^{(2)}$ is computed as follows:

$$d^{(1)}(1,1) = \min\left(d^{(0)}(1,1), d^{(0)}(1,1) + d^{(0)}(1,1)\right)$$
$$d^{(1)}(1,2) = \min\left(d^{(0)}(1,2), d^{(0)}(1,1) + d^{(0)}(1,2)\right)$$
$$d^{(1)}(2,1) = \min\left(d^{(0)}(2,1), d^{(0)}(2,1) + d^{(0)}(1,1)\right)$$
$$d^{(1)}(2,2) = \min\left(d^{(0)}(2,2), d^{(0)}(2,1) + d^{(0)}(1,2)\right)$$

$$d^{(2)}(1,1) = \min\left(d^{(1)}(1,1), d^{(1)}(1,2) + d^{(1)}(2,1)\right)$$
$$d^{(2)}(1,2) = \min\left(d^{(1)}(1,2), d^{(1)}(1,2) + d^{(1)}(2,2)\right)$$
$$d^{(2)}(2,1) = \min\left(d^{(1)}(2,1), d^{(1)}(2,2) + d^{(1)}(2,1)\right)$$
$$d^{(2)}(2,2) = \min\left(d^{(1)}(2,2), d^{(1)}(2,2) + d^{(1)}(2,2)\right)$$

  Note that result of $d^{(2)}$ depends on the result of $d^{(1)}$ and therefore cannot be computed in parallel with the computation of $d^{(1)}$.

- **Phase 2:** Pivot-row and pivot-column blocks

  In the K-th iteration, it computes all $D_{(h,K)}^{(K\times B)}$ and $D_{(K,h)}^{(K\times B)}$ where $h \neq K$.

  The result of pivot-row/pivot-column blocks depend on the result in Phase 1 and itself

  For instance, in the 1st iteration, the result of $D_{(1,3)}^{(2)}$ depends on $D_{(1,1)}^{(2)}$ and $D_{(1,3)}^{(0)}$:

$$d^{(1)}(1,5) = \min\left(d^{(0)}(1,5), d^{(2)}(1,1) + d^{(0)}(1,5)\right)$$
$$d^{(1)}(1,6) = \min\left(d^{(0)}(1,6), d^{(2)}(1,1) + d^{(0)}(1,6)\right)$$
$$d^{(1)}(2,5) = \min\left(d^{(0)}(2,5), d^{(2)}(2,1) + d^{(0)}(1,5)\right)$$
$$d^{(1)}(2,6) = \min\left(d^{(0)}(2,6), d^{(2)}(2,1) + d^{(0)}(1,6)\right)$$

$$d^{(2)}(1,5) = \min\left(d^{(1)}(1,5), d^{(2)}(1,2) + d^{(1)}(2,5)\right)$$
$$d^{(2)}(1,6) = \min\left(d^{(1)}(1,6), d^{(2)}(1,2) + d^{(1)}(2,6)\right)$$
$$d^{(2)}(2,5) = \min\left(d^{(1)}(2,5), d^{(2)}(2,2) + d^{(1)}(2,5)\right)$$
$$d^{(2)}(2,6) = \min\left(d^{(1)}(2,6), d^{(2)}(2,2) + d^{(1)}(2,6)\right)$$

- **Phase 3:** Other blocks

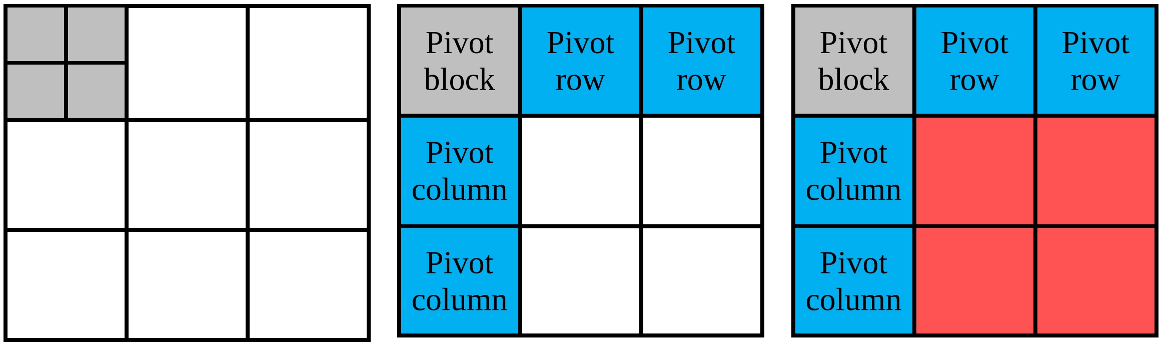
  In the K-th iteration, it computes all $D_{(h_1,h_2)}^{(K\times B)}$ where $h_1, h_2 \neq K$.

  The result of these blocks depend on the result in Phase 2 and itself.

  For instance, in the 1st iteration, the result of $D_{(2,3)}^{(2)}$ depends on $D_{(2,1)}^{(2)}$ and $D_{(1,3)}^{(2)}$:

$$d^{(1)}(3,5) = \min\left(d^{(0)}(3,5), d^{(2)}(3,1) + d^{(2)}(1,5)\right)$$
$$d^{(1)}(3,6) = \min\left(d^{(0)}(3,6), d^{(2)}(3,1) + d^{(2)}(1,6)\right)$$
$$d^{(1)}(4,5) = \min\left(d^{(0)}(4,5), d^{(2)}(4,1) + d^{(2)}(1,5)\right)$$
$$d^{(1)}(4,6) = \min\left(d^{(0)}(4,6), d^{(2)}(4,1) + d^{(2)}(1,6)\right)$$

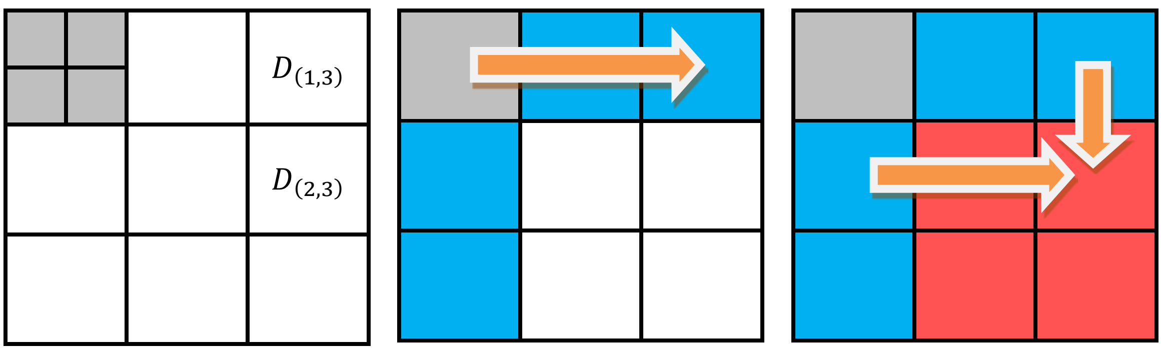
$$d^{(2)}(3,5) = \min\left(d^{(1)}(3,5), d^{(2)}(3,2) + d^{(2)}(2,5)\right)$$
$$d^{(2)}(3,6) = \min\left(d^{(1)}(3,6), d^{(2)}(3,2) + d^{(2)}(2,6)\right)$$
$$d^{(2)}(4,5) = \min\left(d^{(1)}(4,5), d^{(2)}(4,2) + d^{(2)}(2,5)\right)$$
$$d^{(2)}(4,6) = \min\left(d^{(1)}(4,6), d^{(2)}(4,2) + d^{(2)}(2,6)\right)$$



(a) Phase 1 (b) Phase 2 (c) Phase 3

Figure 2: The 3 phases of blocked FW algorithm in the 1st iteration

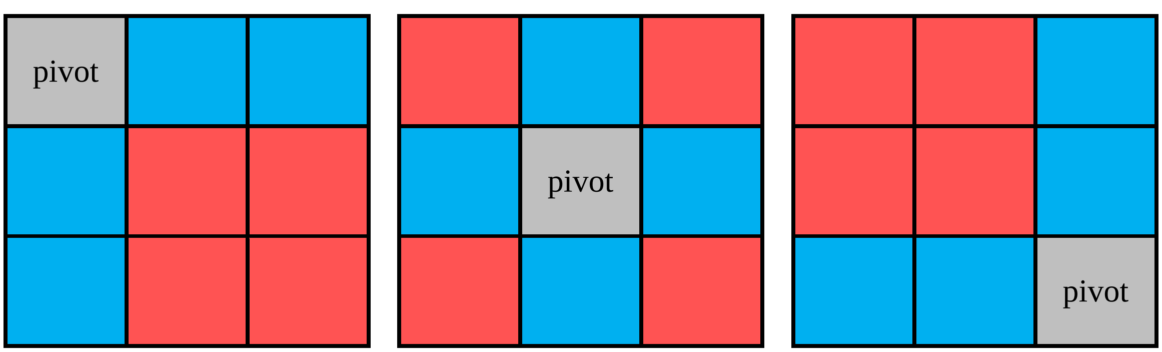
The computations of $D^{(2)}_{(1,3)}, D^{(2)}_{(2,3)}$ and its dependencies are illustrated in Figure 3.



(d) Phase 1 (e) Phase 2 (f) Phase 3

Figure 3: Dependencies of $D^{(2)}_{(1,3)}, D^{(2)}_{(2,3)}$ in the 1st iteration

In this particular example where $N = 6$ and $B = 2$, we will require $\lceil N/B \rceil = 3$ rounds.



(a) Round 1 (b) Round 2 (c) Round 3

Figure 4: Blocked FW algorithm in each iteration

3 INPUT / OUTPUT FORMAT

1. Your program is required to read an input file, and generate output in another file.
2. Your program accepts 2 input parameters. They are:

   i、 (String) the input file name
   ii、 (String) the output file name
   iii、 (Integer) the blocking factor

   Make sure users can assign test cases through command line. For instance:

   ```
   $  ./executable in_file out_file 32
   ```

   TAs will judge your program as follows:

   ```
   $  diff -b out_file answer
   ```

3. The 1st line of an input test case consists of 2 integers ***N*** $(1 \leq N \leq 10000)$ and ***M*** $(0 \leq M \leq 10^9)$ separated by a single space, which represents number of vertices and number of edge weight assignments respectively.

   Each of the following M lines consists of 3 integers ***i***, ***j*** and ***W*** $(i \neq j)$, separated by a single space between any two numbers.

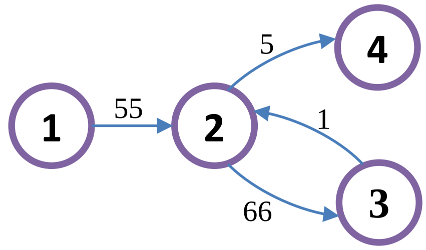
   - ***i*** represents the index of the source vertex $(1 \leq i \leq N)$
   - ***j*** represents the index of the destination vertex $(1 \leq j \leq N)$
   - ***W*** represents the distance (weight of edge) from vertex i to vertex j $(0 \leq W \leq 100)$

   Edges which are not listed in the input file do not exist in the graph. That is, for all $i \neq j$, if edge(i, j) does not show up in the input at all, vertex i does not have an edge to vertex j. But since we are dealing with a ***directed*** graph, this does NOT imply that edge(j, i) is also non-existent.

   Besides, if there are re-assignments of an edge, please follow the latest one.

```
[s104012345@pp01 ~]# cat testcase
4 4
1 2 55
2 3 66
3 2 1
2 4 5
```

(a) Content of a sample input file



(b) The corresponding graph

Figure 5: Sample Input

4. For output file, list the shortest-path distance of all vertex pairs.

   Assume *N* represents total number of vertices, the output file should consists of *N* lines, each line consists of *N* numbers and separate them by a single space.

   The number at the ith line and the jth column is the shortest-path distance from the ith vertex to the jth vertex if there is a path; otherwise, the corresponding output should be **INF**.

```
[s104012345@pp01 ~]# cat output
0 55 121 60
INF 0 66 5
INF 1 0 6
INF INF INF 0
```

Figure 6: Sample output

The sample test cases are provided in **/home/ipl2017/shared/hw4** on pp31.

4 Working Items

You are required to implement 2 versions of blocked Floyd-Warshall algorithm under the given restrictions.

1. Single GPU

- Implement blocked APSP algorithm as described in Section 2.
- The main algorithm should be implemented in CUDA C/C++ kernel functions.
- Achieve better performance than sequential Floyd-Warshall implementation.

2. **Multi GPUs implementation with OpenMP**
   - The restrictions of single-GPU version still hold.
   - Able to utilize multiple GPUs available on single node.
   - Achieve better performance than single GPU version.
3. **Makefile**
   Please refer to the example in **/home/ipl2017/shared/hw4** on **apolloGPU**.
   Don't modify execution file name(HW4_cuda.exe, HW4_openmp.exe) in sample Makefile.
4. **README**
   You should specify your best block factor in README, TAs will use this configuration to test your performance.
   Please refer to the example in **/home/ipl2017/shared/hw4** on **apolloGPU**.

5 Optimization Hints

- Shared memory
- Streaming
- Resolve bank conflicts
- Dynamic load-balancing (for openMP and MPI)

6 Grading

1. **Correctness** (70%)
   i、 [50%] Single-GPU
   ii、 [20%] Multi-GPU implementation with OpenMP
2. **Performance** (20%)
   - Performance is measured by the execution time of your program using 'time' Linux command.
   - Points are giving according to the performance ranking of your program among all the students.
3. **Demo** (10%)
   - Each student is given 10 minutes to explain your implementation followed by some questions from TA.
   - No debugging or code modification is allowed during the demo.

- Points are given according to your understanding and explanation of your code, and your answers of the TA questions.

7 REMINDER

1. Please upload the following files to **HW_submission/HW4** directory on **apolloGPU** under your home directory before **8/3 8:00AM** (**The folder will be locked after deadline**)

   i、 HW4_{student-ID}_cuda.cu

   ii、 HW4_{student-ID}_openmp.cu

   iii、 Makefile

   iv、 README

   Make sure your compile script can execute correctly and your code has no compile error before you upload your homework.

2. We provide sample code seq_FW.cpp and block_FW.cpp in **/home/ipl2017/shared/hw4** on **apolloGPU.**

3. Since we have limited resources for you guys to use, please start your work ASAP. Do not leave it until the last day!

4. **0 will be given to cheater** (even copying code from the Internet), but discussion on code is encouraged.

5. Asking questions through iLMS or by email are welcomed!